

SHORT COMMUNICATION

ON SECONDARY CIRCULATION, HELICAL MOTION AND ROZOVSKII-BASED ANALYSIS OF TIME-AVERAGED TWO-DIMENSIONAL VELOCITY FIELDS AT CONFLUENCES

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ABSTRACT

In this response to Lane *et al.* (1999) 'Time averaged flow structure in the central region of a stream confluence: a discussion', *Earth Surface Processes and Landforms*, **24**, 361–367 we show that our method for decomposing the cross-stream velocity field into components associated with primary and secondary velocities, as defined by Rozovskii, is not flawed or misleading. Instead, it yields valuable information that illustrates the contribution of secondary circulation and, by inference, helical motion to the pattern of cross-stream flow at confluences. Lane *et al.*'s concern about our methodology can be attributed to their failure to distinguish clearly and consistently between secondary circulation and cross-stream discharge, a distinction that is central to our method of analysis, and to their inappropriate comparisons of velocity fields for the different frames of reference associated with secondary circulation and cross-stream discharge. Copyright 1999 John Wiley & Sons, Ltd.

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INTRODUCTION

Perhaps the worst fate any published work can suffer is for it to be ignored. We therefore are flattered that a group of scholars as distinguished as Lane *et al.* (1999) have sufficient interest in our paper to develop a commentary on it. Conversely, we are dismayed that their interest hinges on an attempt to characterize our method of analysis as flawed and misleading. We obviously disagree with this assessment and contend that their criticisms derive from misconceptions about the difference between secondary circulation and cross-stream discharge, a distinction that is central to our method of analysis, and from inappropriate comparisons of velocity fields for different frames of reference.

The characterization of our work as flawed and misleading is not an accusation we take lightly. The seriousness of this accusation necessitates a detailed rejoinder to defend the quality of our research. In our estimation, a method is flawed if it: (1) introduces information into the analysis that is not embodied in the original data; (2) is applied in an arbitrary, subjective and inconsistent manner; (3) clearly violates established standards, assumptions, theories or rules underlying the method; or (4) yields outcomes that cannot be reproduced by other investigators. A method usually is viewed as misleading if it generates information that is irrelevant to the defined objectives of the research or if the information derived from application of the method is interpreted in an improper manner, i.e. in a manner that does not conform with accepted background knowledge about patterns of data an inferred process could possibly produce. As we demonstrate in this response, none of these conditions applies to our method of analysis.

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THE CRITICISMS

The criticisms of Lane *et al.* (1999) focus mainly on our use of Rozovskii's (1957, pp. 199–201) procedure to decompose cross-stream velocity data into components associated with primary (v_{py}) and secondary (v_{sy}) velocities. Not only are they critical of this decomposition, but they also doubt the validity of the inferences derived from it, especially those based on patterns of v_{sy} . Before we defend our methodology, we would like to point out to casual readers of this discussion that the Rozovskii-based decomposition constitutes only one component of the total body of results presented in our paper. Much of what we say in the paper has no relation to the Rozovskii-based analysis. The main issue this type of analysis does have relevance for concerns whether or not helical motion exists within the complex flow field at confluences.

The criticisms of our use of the Rozovskii method can be distilled into three main issues: (1) the extent to which patterns of secondary circulation are an artefact of the methodology; (2) whether or not the Rozovskii procedure and our modification of it provide an appropriate frame of reference for analysing confluence flows; and (3) problems associated with using patterns of v_{sy} to infer the magnitude of vertical velocities. The following discussion focuses mainly on these three issues. Lane *et al.*'s (1999) discussion of intermittence and averaging is not specific to our study, but instead applies to any investigation based on time-averaged velocity data, including three-dimensional (3D) data, using any frame of reference for displaying the information. We do not dispute the importance of exploring the issues raised in the discussion of intermittency, but such an exploration is merely a logical extension of, and complement to, investigations based on time-averaged data, not an inherently superior approach to research on confluences.

SOME FUNDAMENTAL MISCONCEPTIONS

Any criticism of a particular method should be based on a sound understanding of the conceptual framework underlying it and of the information produced by the technique. All three of the main criticisms by Lane *et al.* (1999) result from their failure to distinguish clearly and consistently between secondary circulation and cross-stream discharge, which are defined within different frames of reference, and their inappropriate comparison of the velocity fields associated with these different frames of reference. For example, Lane *et al.* (1999) claim that Dietrich and Smith (1983) recognized that 'secondary circulation involves net cross-stream and downstream transfer of mass and momentum in the form of a helix, rather than as a closed circulation cell.' A careful reading of Dietrich and Smith (1983, p. 1184) and Dietrich (1987, pp. 191, 194), however, shows that they correctly acknowledged the difference between secondary circulation, as defined by Rozovskii (1957), and cross-stream discharge. Secondary circulation, by definition, involves zero net lateral discharge relative to the primary flow direction (Chang, 1988, p. 301), whereas cross-stream discharge, defined either by the path of the channel centreline (Rhoads and Kenworthy 1995, 1998; Rhoads 1996) or on the basis of continuity (Dietrich and Smith, 1983), can involve net transverse fluxes of water. Dietrich (1987, p. 194) also points out that the plane of secondary circulation in meander bends is often curved, a phenomenon that our analysis suggests is the usual case in confluences.

Although meander research has moved away from using the Rozovskii method, this shift has nothing to do with any inherent 'flaws' or 'shortcomings' of this method, but was prompted by the development of Smith and McLean's (1984) mathematical model of meandering, which was based on a downstream and cross-stream coordinate system in relation to the pattern of channel curvature. The Rozovskii method does not in any way 'exaggerate' the amount of secondary circulation as argued by Lane *et al.* (1999). It cannot do so because the method *defines* the frame of reference of secondary circulation. On the other hand, lateral discharge may exist in the cross-stream direction *because* this direction corresponds to a different frame of reference. In other words, the same flow can be characterized by lateral discharge in the cross-stream direction and secondary circulation (zero net secondary discharge) in the Rozovskii frame of reference. Only in the case where each depth-averaged velocity vector is oriented perpendicular to the cross-section will the cross-stream and secondary-circulation velocity fields be identical.

Lane *et al.*'s (1999) concern about exaggeration of secondary circulation arises from an inappropriate comparison of velocity components in two different frames of reference. They implicitly privilege the cross-

stream frame of reference and then denounce the Rozovskii method as 'flawed' because it yields velocity information that differs from that for their preferred frame of reference (e.g. figure 1 in Lane *et al.*, 1999). Such an approach is clearly inappropriate – it amounts to a comparison of apples and oranges.

Any attempt to compare velocity components for different frames of reference must utilize an appropriate coordinate transformation scheme (Aris, 1962). Our decomposition method is an example of an appropriate transformation scheme for identifying the contributions of two components defined in the Rozovskii frame of reference – secondary circulation and skewing of the primary flow – to the cross-stream velocity field. As we show in our paper, the sum of these two components (v_{sy} and v_{py}) equals the cross-stream velocities. Thus, differences between *one* of these components and the complete cross-stream velocity field are to be expected and it is not appropriate either to use the v_{sy} to estimate the magnitudes of vertical velocities in the cross-stream plane or to compare the results of this inappropriate analysis with estimates of vertical velocities derived from analysis of the complete 3D flow field (e.g. Lane *et al.*, 1999). It must also be pointed out that Lane *et al.*'s (1999) implicit privileging of the cross-stream frame of reference relative to the Rozovskii frame of reference has no inherent justification. The appropriateness of a particular frame of reference depends solely on the objectives of the analysis; no frame of reference is *inherently* superior to another (Aris, 1962, pp. 3–6). The cross-stream frame of reference and Rozovskii frame of reference merely illustrate different components of the same underlying physical quantities, i.e. velocity vectors, and thus one of these frames of reference is not more 'accurate', 'correct' or 'realistic' than the other. The following discussion illustrates how these fundamental misconceptions nullify all of Lane *et al.*'s criticisms of our work and also emphasizes the value of our method for analysing the complex structure of flow at confluences and other fluvial environments.

HELICAL MOTION AND THE ROZOVSKII METHOD

Past studies in fluvial geomorphology have inferred helical motion from two-dimensional (2D) velocity data based on two criteria: (1) that the orientations of velocity vectors vary systematically over depth with the maximum deviation angles occurring between the near-surface and near-bed flows; and (2) that this pattern of vector variation exists at more than a single vertical (e.g. Prus-Chacinski, 1954; Yen, 1965; Jackson, 1975; Hooke, 1975; Hickin, 1978; Alphen *et al.*, 1984; Thorne and Furbish, 1995). This approach is not restricted to the analysis of flow in meandering rivers, but also can be used to infer the presence and strength of helical motion in straight channels with curved flow streamlines (Leopold, 1982; Nelson and Smith, 1989). The Rozovskii method is an information-rich variant of this general method for detecting helicity within the flow. It reveals the secondary velocities induced by divergence of near-bed and near-surface velocity vectors and also indicates whether these secondary velocities vary systematically over depth. As we mention in our paper (Rhoads and Kenworthy, 1998, p. 176), helical motion will produce a systematic pattern of vector orientations, and thus secondary circulation, over depth at adjacent verticals. Because the secondary velocities (v_s) represent the component of the flow at each point in a vertical directed orthogonal to the orientation of the depth-averaged vector at that vertical, they provide a measure of the strength of secondary circulation in terms of velocity rather than in terms of angles of deviation.

Nothing about the Rozovskii procedure guarantees that a pattern indicative of large-scale helicity will be produced by application of the procedure to the data. The method only reveals information actually embodied in the velocity-vector data; it does not 'manufacture' patterns that are not contained within these data. It simply is not true that the method 'will always produce an apparent circulation cell or cells whenever there is flow divergence or convergence' (Lane *et al.*, 1999). The flow may be converging or diverging, but if velocity vectors over depth at each vertical have the same orientation within this diverging or converging flow, the secondary circulation will be zero over the entire cross-section. Moreover, if vectors are oriented randomly over depth, or if each vertical has a different pattern of vector orientations over depth, the pattern of secondary circulation, while still constrained by the zero net discharge requirement, will not conform with the pattern produced by large-scale ('inter-vertical') helicity.

The identification of large-scale helical motion on the basis of patterns of secondary velocities can be subjective (this problem also applies to the identification of helicity in the cross-stream plane), but, in contrast

to previous studies, we have attempted to add rigour to this procedure by defining specific criteria for distinguishing large-scale helical motion from weak 'cells' associated with random noise in the velocity measurements or non-systematic variations in vector orientations over depth (Rhoads and Kenworthy, 1998, p. 184). This procedure provides an objective basis for determining whether coherent patterns of secondary circulation exist in a curved, but consistently defined flow 'face' oriented perpendicular to the local orientations of the depth-averaged velocity vectors. Lane *et al.* (1999) may choose to disagree with our criteria for defining helical motion, but such disagreement would be merely a matter of opinion, not of right versus wrong.

THE ROZOVSKII FRAME OF REFERENCE

Lane *et al.* (1999) are concerned by what they characterize as 'an element of circularity' (we assume no pun is intended) in analysis of the flow using the Rozovskii method. Such a concern is unfounded. Because the depth-averaged velocity vectors are oriented tangentially to the depth-averaged streamlines, centrifugal forces associated with any curvature of these streamlines will be oriented perpendicular to the depth-averaged vector (Rozovskii, 1957, p. 201). Differences in the strength of centrifugal forces over depth in a curving flow subject to a transverse pressure gradient can induce helical motion, as documented rather conclusively in studies of meandering rivers (Jackson, 1975; Hooke, 1975; Hickin, 1978; Alphen *et al.*, 1984). This helical motion will be expressed as systematic deviations of individual velocity vectors at several adjacent verticals from the orientation of the depth-averaged vectors. Thus, the frame of reference underlying the Rozovskii method has a justifiable theoretical basis.

Lane *et al.* (1999) also are bothered by the concept of a 'closed' loop of secondary circulation. The identification of loops of circulation is common in fluid mechanics (White, 1986, pp. 447–448; Tritton, 1988, pp. 84–85). Such loops are not closed in the sense of an isolated zone of recirculating fluid (a silly notion in a flow moving predominantly in the downstream direction), but rather indicate relative motion around a frame of reference moving with the flow streamlines. Although loops of circulation indicate the presence of vorticity in the flow (Tritton, 1988), they certainly do not imply that a specific parcel of water actually moves around the path of circulation *within a cross-section*. The paths followed by the fluid, at least in a time-averaged sense, are indicated by the orientations of the velocity vectors.

We acknowledge the importance of the cross-stream velocity field, especially for interpreting patterns of sediment movement or scour/fill at a fixed cross-section, but believe our method combines the best of both worlds. Our decomposition not only reveals the patterns of velocity in the downstream and cross-stream directions, but also shows how the cross-stream velocities at fixed cross-sections change as the strength and coherence of secondary circulation in a curvilinear plane, defined by the orientation of the depth-averaged velocity vectors, changes with flow stage. For example, low-momentum-ratio, high-stage flows at the Copper Slough–Kaskaskia River confluence exhibit relatively strong secondary circulation, resulting in flow divergence near the bed in the cross-stream plane at cross-section A (Rhoads, 1996). During low-stage, low-momentum-ratio events, the strength of secondary circulation is not always sufficient to produce divergent near-bed flow in the cross-stream plane at cross-section A (Rhoads and Kenworthy, 1998). Despite Lane *et al.*'s (1999) objections, no inherent problems are generated by rotating the secondary velocities (v_s) back onto the cross-stream plane (v_{sy}). Doing so merely allows one to conduct a type of analysis that yields rich information about the structure of the flow.

Although the downstream velocity field could also be decomposed into components associated with v_{px} and v_{sx} , values of v_{sx} generally are a small fraction of the v_{px} , indicating that secondary circulation does not contribute in a substantial way to the magnitudes of downstream velocities. Decomposition of the downstream velocity field is straightforward, however, and does not pose any serious conceptual, theoretical or analytical problems.

A disadvantage of examining the cross-stream velocity field only is that such analysis fails to reveal explicitly the full extent of spiral motion within the flow and how changes in this spiral motion can affect flow in other frames of reference, such as the velocity field in the cross-stream plane. For example, Dietrich (1987) acknowledges that the interaction of centrifugal and pressure gradient forces in meandering rivers still

generates secondary circulation over the point-bar surface, but that forces associated with topographic steering of the flow orient all vectors, including near-bed ones, outward toward the pool *in the cross-stream plane*. Thus, flow over the point bar is still part of the overall 3D pattern of helicity within the bend (see Dietrich, 1987, figure 89), even though helicity over the point bar is not evident in the cross-stream plane. Using our method of analysis, one could decompose continuity-defined, cross-stream velocity fields in meander bends to illustrate the contribution of secondary circulation in the Rozovskii frame of reference to the overall pattern of cross-stream discharge and to examine how increases in the strength of the cross-stream component of secondary circulation over the point bar with increases in stage eventually produce apparent helical flow over the point-bar surface in the cross-stream plane (cf. Thorne *et al.*, 1985; Dietrich, 1987, pp. 195–196).

Application of the Rozovskii procedure is especially valuable at confluences where the orientation of depth-averaged velocity vectors varies considerably not only along the channel, but also across any particular planar ‘slice’ through the flow. The capacity of the Rozovskii method to readily accommodate local variability in the orientation of depth-averaged velocity vectors can be useful for identifying the presence of helicity within a flow that does not follow the path of channel boundaries and does not move in a uniform direction from one side of the channel to another. We have contended that as confluent flows collide and mutually deflect one another, a complex pressure gradient field will be established to counterbalance the depth-averaged curvature of flow streamlines. In this general sense, the turning of the flow at confluences should be *similar* to flow at meander bends, but we have never claimed that confluences will be ‘just like’ meander bends as stated by Lane *et al.* (1999). The production of streamwise vorticity through skewing of the mean shear by a transverse pressure gradient has the potential to generate helical motion within the confluence, but this process is likely to be more complex at confluences than at meander bends given the variety of geometric configurations of confluences and the dependence of pressure-gradient fields on the dynamic interaction between the confluent flows, including the shear layer between these flows, rather than on the effects associated with a solid boundary (Rhoads and Kenworthy, 1998, p. 187). The Rozovskii procedure provides a way of identifying the presence of helical motion within a flow where the curvature of streamlines exhibits considerable spatial variability (Rozovskii, 1957, pp. 199–203). We agree that any developing 3D helicity can only be examined in a spatially distributed context (over several cross-sections) – a point that Lane *et al.* (1999) emphasize in their arguments and that is consistent with our inferences about the spatial development and reorientation of helical motion, from its initiation within the confluence to its eventual decay within the downstream channel (Rhoads, 1996, pp. 511–514; Rhoads and Kenworthy, 1998, pp. 187–189).

THE VERTICAL-VELOCITY ‘PROBLEM’

Lane *et al.* (1999) devote considerable attention to an analysis whereby they compute magnitudes of vertical velocities based on values of v_{sy} . We have little to say about this analysis, other than that we agree it is inappropriate for analysing the magnitudes of vertical velocities. We never performed such an analysis ourselves, nor would we attempt to do so. Any analysis aimed at computing the magnitude of vertical velocities would have to be based on the complete 2D velocity field, not on values of v_{sy} alone. As mentioned previously, the misguided analysis of Lane *et al.* (1999) reflects a basic misunderstanding of the relation between secondary circulation and cross-stream discharge.

The purpose of our v_{sy} analysis was merely to identify the presence and extent of large-scale helical cells within a converging flow. Inferences were made only about the probable location of vertical motion within the fluid, not about the absolute magnitude of this motion. Again, it must be kept in mind that the extent of secondary circulation revealed by our methodology conforms with the lateral extent of systematic patterns of deviation of individual velocity vectors about the depth-averaged vectors – a pattern typical of flow experiencing helical motion. Inferences about the location of downwelling fluid were not based solely on the pattern of v_{sy} , but also on information about the location and pattern of distortion of the mixing interface between the flows, the pattern of downstream isovels, and the net cross-stream convergence of water within the centre of the confluence. We believe our inferences are based on sound reasoning, but fully acknowledge

(Rhoads and Kenworthy, 1998, pp. 176, 190) that rigorous evaluation of these inferences will require the acquisition of 3D velocity data. We are prepared to be mistaken about inferences concerning 3D fluid motion, but are confident that our analysis accurately depicts the 2D time-averaged structure of the flow and that any discrepancies between inferences derived from 2D and 3D information will be a function of the complexity of confluences, not of an inherent 'flaw' in our methodology.

CONCLUSION

All of Lane *et al.*'s (1999) main criticisms can be attributed to their failure to distinguish clearly and consistently between cross-stream discharge and secondary circulation, a standard distinction that is central to our method of analysis. This laxity leads to inappropriate comparisons of velocity fields for different frames of reference. Their criticisms thus are misguided and do not constitute a set of valid claims about some inherent errancy of our methodology.

The arguments presented in this paper show that our method of decomposing the time-averaged cross-stream velocity field into components associated with primary and secondary velocities, as defined by Rozovskii: (1) does not introduce information into the analysis that is not embodied in the original data; (2) follows a well-defined set of rules that is consistent with theoretical concepts and methodological protocols in river hydraulics and fluid dynamics; and (3) yields reproducible results and can be applied in a flexible manner over a wide range of hydrodynamic settings in rivers. Moreover, the analysis generates information directly relevant to the defined objectives of our research – to explore the influence of coherent patterns of secondary circulation and, by inference, helical motion on the pattern of cross-stream velocity at fixed cross-sections as flow stage varies. We have also established a set of procedures for interpreting our data that is more rigorous than protocols used in previous studies, yet conforms with established theory about the patterns of data certain flow processes, especially helical motion, could possibly produce. Our method of analysis thus is not flawed or misleading, but instead yields valuable information about the nature of 2D time-averaged velocity data that, within the constraints associated with 2D data, provides a reasonable basis for making inferences about the possible 3D structure of flow at confluences. We recommend that investigators engaged in similar research consider using our approach, along with other related methods, to explore the richness of information contained in 2D velocity data, the scope for which our method is intended.

Many of the points Lane *et al.* (1999) raise in the latter part of their paper about the future of research on confluences we wholeheartedly agree with, but feel are irrelevant to the commentary at hand, which concerns the present (or by now the recent past) not the future. Certainly, an improved understanding of confluence dynamics will emerge through integrated field and modelling efforts based on 3D velocity information. We have repeatedly acknowledged the need for 3D data in our own work. But if by emphasizing the need for such studies Lane *et al.* (1999) apparently mean to imply that investigations based on 2D data should not have been conducted in the first place because they may produce fallible results, we would emphatically disagree. Science is a process, not an enterprise that culminates in certified truth. Each new study, except perhaps for those that are flawed according to the criteria defined above, contributes to this process by pushing forward the frontiers of inquiry. Every investigator must decide for him- or herself whether alternative approaches to the same research problem are complementary or adversarial – a judgment that is a matter of opinion (or philosophy), not 'right' versus 'wrong', 'truth' versus 'falsehood' or 'correct' versus 'flawed'.

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